UNIVERSITI SAINS MALAYSIA

First Semester Examination
Academic Session 2007/2008

October/November 2007

EUM 111 – ENGINEERING MATHEMATICS
[Matematik Kejuruteraan]

Duration: 3 hours
[Masa: 3 jam]

Please check that this examination paper consists of SEVEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi TUJUH muka surat bercetak sebelum anda memulakan peperiksaan ini].

This paper contains SIX questions.

[Kertas soalan ini mengandungi ENAM soalan].

Instructions: Answer **FIVE** (5) questions. If a candidate answer more than five questions, only the first five answered will be examined and awarded marks.

*[**Arahan:** Jawab **LIMA** soalan. Jika calon menjawab lebih daripada lima soalan hanya lima soalan pertama mengikut susunan dalam skrip jawapan akan diberi markah].*

Answer to any question must start on a new page.

[Mulakan jawapan anda untuk setiap soalan pada muka surat yang baru].

You may answer a question either in bahasa Malaysia or in English.

[Anda dibenarkan menjawab soalan sama ada dalam bahasa Malaysia atau bahasa Inggeris].

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1. (a) Selesaikan persamaan pembezaan berikut.

Solve the following differential equation.

(i) $\dfrac{dy}{dx} = \dfrac{5x+4y}{2x-y}$

(ii) $y\dfrac{dy}{dx} = e^{x+2y}\sin x$

(10 marks)

(b) Fungsi $f(t)$ takrifkan sebagai

A function $f(t)$ is defined by

$$f(t) = \sin t\,, \quad 0 < t < \pi$$

(i) Dapatkan kembangan siri Fourier kosinus separuh julat untuk $f(t)$.

Obtain a half range Fourier cosine series expansion for $f(t)$.

(ii) Apakah nilai penumpuan siri ini apabila $t = 1$ dan $t = \pi$?

To what value does this series converge when $t = 1$ and $t = \pi$?

(10 marks)

2. (a) Dengan menggunakan kaedah pemisah pemboleh ubah, selesaikan persamaan haba

Using the separation of variable method, solve the heat conduction equation

$$\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}$$

yang memenuhi syarat
that fulfilled the condition

$$u(0,t) = u(3,t) = 0 \text{ and } u(x,0) = 4\sin 4\pi x - 3\sin 8\pi x + 2\sin 10\pi x$$

(12 marks)

(b) Suatu perintang R dan induktor L disambung secara siri dengan $E(t)$ adalah voltan yang dikenakan terhadap litar. Persamaan pembezaan yang menerangkan aliran elektrik $i(t)$ diberi sebagai

A resistor R and an inductor L are connected in series with $E(t)$ being the voltage impressed on the circuit. The differential equation that describe the current $i(t)$ is as given below

$$L\frac{di}{dt} + Ri = E(t) \ ,$$

Jika voltan 30 volt dikenakan kepada litar LR dengan jumlah rangsangan aliran elektrik 0.1 Henry dan rintangan ialah 50 ohms, dapatkan $i(t)$ jika $i(0) = 0$. Tentukan aliran elektrik jika $t \to \infty$.

If a voltage of 30 volt is applied to the LR circuit in which the inductance is 0.1 Henry and the resistance is 50 ohms, find $i(t)$ if $i(0) = 0$. Determine the current as $t \to \infty$.

(8 marks)

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3. (a) Dapatkan dua penyelesaian siri kuasa untuk persamaan pembezaan berikut:

Find the two power series solutions for the following differential equation:

$$(x-1)y'' + y' = 0$$

(8 marks)

(b) Dapatkan Jelmaan Laplace untuk fungsi
Find the Laplace Transform of the function

$$f(t) = \delta(t-2) + \delta(t-4)$$

Seterusnya, selesaikan persamaan pembezaan berikut mengikut nilai awal yang diberi.

Hence, solve the following differential equation subjected to the indicated initial conditions.

$$\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 6y = e^t + \delta(t-2) + \delta(t-4); \quad y(0) = 1,\ y'(0) = 0$$

(12 marks)

4. (a) Diberi sistem persamaan pembezaan linear berikut:

Given the following system of linear differential equations:

$$\frac{dx_1}{dt} = x_1 - 2x_2 + 2x_3$$
$$\frac{dx_2}{dt} = -2x_1 + x_2 - 2x_3$$
$$\frac{dx_3}{dt} = 2x_1 - 2x_2 + x_3$$

Nyatakan dalam bentuk $x' = Ax$ dengan $x = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T$.

Carikan nilai eigen dan vektor eigen sepadan untuk A. Kemudian, tentukan penyelesaian am untuk sistem itu.

Express these in the form $x' = Ax$ *where* $x = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T$

Find the eigenvalues and the corresponding eigenvectors of A. *Then, determine the general solution of the system.*

(12 marks)

(b) Dengan menggunakan kaedah Operator D, selesaikan persamaan pembezaan berikut:

Using Operator D method, solve the following differential equation:

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2e^x \cos\left(\frac{x}{2}\right)$$

(8 marks)

5. (a) (i) Dapatkan fungsi pelengkap untuk persamaan pembezaan berikut:

Find the complementary functions for the following differential equations:

$$\frac{d^2y}{dx^2} + y = 0$$

(ii) Seterusnya, dengan menggunakan jawapan bahagian (i), selesaikan persamaan pembeza.

Hence, using the answers from part (i) above, solve the differential equation.

$$\frac{d^2y}{dx^2} + y = \cos^2 x$$

(10 marks)

(b) Selesaikan sistem persamaan linear berikut dengan kaedah 'Doolittle' LU.

Solve the following system of linear equations using the Doolittle LU method.

$$2x + -3 + 3z = -2$$
$$6x - 8y + 7z = -3$$
$$-2x + 6y - z = 3$$

(6 marks)

(c) Buktikan bahawa $u(x,t) = e^{-8t}\sin 2x$ adalah penyelesaian untuk persamaan haba $\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x}$ dengan $u(0,t) = u(\pi,t) = 0;\ u(x,0) = \sin 2x$

Prove that $u(x,t) = e^{-8t}\sin 2x$ *is the solution of the heat conduction equation* $\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x}$ *with* $u(0,t) = u(\pi,t) = 0;\ u(x,0) = \sin 2x$

(4 marks)

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6. (a) Dapatkan sama ada Jelmaan Laplace atau songsangan Jelmaan Laplace berikut:

Find either the Laplace Transform or the inverse Laplace Transform as indicated:

(i) $L\{\sin 4t \cos 3t\}$

(ii) $L\{t^2 H(t-3)\}$

(iii) $L^{-1}\left\{\dfrac{(1-e^{-s})(1+e^{-2s})}{s^2}\right\}$

(iv) $L\{f(t)\}$ where $f(t) = \begin{cases} \sin t & 0 \le t \le \pi \\ 0 & \pi \le t \le 2\pi \end{cases}$

and $f(t) = f(t+2\pi)$ (8 marks)

(b) Fungsi berkala $f(t)$ diberi sebagai

A periodic function $f(t)$ is given below as

$$f(t) = \begin{cases} t & \left[0, \dfrac{\pi}{2}\right) \\ \dfrac{\pi}{2} & \left(\dfrac{\pi}{2}, \pi\right) \\ \pi - \dfrac{t}{2} & (\pi, 2\pi] \end{cases}$$

dengan

with $f(t) = f(t+2\pi)$

Lakarkan graf untuk fungsi $f(t)$ antara $-\pi$ ke 3π dan dapatkan Siri Fourier tersebut.

Sketch a graph of the function $f(t)$ between $-\pi$ to 3π and find the Fourier Series function of it.

(12 marks)

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